on the deformation of the cylinder. As the ratio R'/R is fairly large in the actual cases considered, the main term expressing the cylinder distortion is proportional to $(1+\sigma)/E$, i. e. to $1/G$ where G is the modulus of rigidity.

Interpreting k as the quotient of the two moduli of rigidity, we now have

$$\begin{aligned}\lambda_A - \lambda_B &= \varphi_A - \varphi_B + \theta_A - \theta_B \\ &= (1-\mathrm{k})\,\varphi_A + \theta_A - \theta_B \\ &= (1-\mathrm{k})\,\lambda_A + (k\theta_A - \theta_B) \qquad (3.5)\end{aligned}$$

determining λ_A in terms of the difference coefficient $\lambda_A - \lambda_B$ established by the balancing procedure, the value of k, and the correction term $(k\theta_A - \theta_B)$.

d) Extension to the use of three materials

In the first series of experiments the material adopted for the comparison assemblies was a form of aluminium bronze, known commercially as "hydurax", the modulus of rigidity of which was lower than that of steel in the ratio 1:1.44. The Poisson's ratio was rather higher than that of steel (see Tab. 1 for further details). It was apparent that a check involving a third material, differing substantially in elastic properties from those used hitherto, would provide a valuable test of the accuracy of the similarity method. An even better check would naturally be provided by a completely independent pair of materials. This latter extension has not so far been found practicable as the choice of materials possessing all the qualities required is limited. It has been found possible, however, to extend the procedure to include three materials, the third being an alloy of tungsten known commercially as "GEC Heavy Metal". This material proved to have a high degree of isotropy and a Poisson's ratio very close to that of the material used for the steel assemblies. The elastic moduli exceed those of steel in about the ratio 1.75:1 and it was of advantage that in this case the comparison should involve a material having a modulus higher than that of steel in contrast to the former comparisons in which the reverse held.

In discussing this extension of the method it will be convenient to refer to the steel, bronze and tungsten assemblies by the initial letters S, B and T respectively. With a group of three materials, the distortion coefficient of any one assembly, say S, may be reached by three different routes, two of them direct – i. e. involving direct comparisons with the other two assemblies B and T – and the other indirect. In the latter procedure the distortion coefficient of one of the other two assemblies, say T, is first determined by applying the similarity principle to B and T, and the coefficient for S is then obtained by simple addition of the difference coefficient for S and T. It is of interest to note that the indirect procedure leads to the distortion coefficient of the assembly chosen, i. e. S, without any appeal to the elastic constants of the material of S. These three derivations are not entirely independent but, since the six independent elastic moduli involve five independent ratios, no one result is in general deducible from the other two. Proceeding on the lines of equation (3.5) and denoting by $\lambda_S \ldots$ the true values of the distortion coefficients, $G_S \ldots$ the moduli of rigidity, $\theta_S \ldots$ the corresponding correction terms given by equation (3.4), λ_{SB} ($= \lambda_S - \lambda_B$) ... the difference coefficients determined by the balancing experiments, and $k_{SB} = G_B/G_S \ldots$, we have for the three possible experimental values, λ'_S, λ''_S and λ'''_S, of λ_S, the equations

$$\lambda'_S\,(k_{BS}-1) = \lambda_{BS} - \theta_B + k_{BS}\,\theta_S \qquad (3.6)$$

$$\lambda''_S\,(k_{TS}-1) = \lambda_{TS} - \theta_T + k_{TS}\,\theta_S \qquad (3.7)$$

for the direct comparisons, and for the indirect

Table 1. *Summary of elastic constants*

Material	Modulus of rigidity (G) (dyn/cm²)*		Young's modulus (E) (dyn/cm²)*		Poisson's ratio (σ) (Ultrasonic pulse method)
	Torsion extensometer method	Ultrasonic pulse method	Extensometer method (mean of results for tension and compression)	Ultrasonic pulse method	
Steel (K 9) (hardened and tempered)	7.86 × 10″	7.92 × 10″	20.6 × 10″	20.5 × 10″	0.295
Aluminium bronze ("hydurax")	5.45 × 10″	5.38 × 10″	14.4_5 × 10″	14.3_3 × 10″	0.333
Tungsten alloy ("GEC heavy metal" – specific gravity 18)	13.5_5 × 10″	14.2_5 × 10″	36.1 × 10″	37.7 × 10″	0.286_5

* 1 dyn/cm² = 0.1 N/m².

$$\lambda'''_S\,(k_{BT}-1) = \lambda_{BT} - \theta_B + k_{BT}\,\theta_T + \lambda_{ST}\,(k_{BT}-1). \qquad (3.8)$$

Transposing these equations and making use of the subsidiary relations

$$\begin{aligned} &k_{BS}\,k_{SB} = 1 \ldots\,; \quad k_{BS}\,k_{ST}\,k_{TB} = 1\,; \\ &\lambda_{BS} = -\lambda_{SB} \ldots\,; \quad \lambda_{BS} + \lambda_{ST} + \lambda_{TB} = 0\,; \end{aligned} \qquad (3.9)$$

we eventually obtain

$$(\lambda'_S - \lambda'''_S)\,(1-k_{SB}) = (\lambda'''_S - \lambda''_S)\,(k_{ST}-1). \qquad (3.10)$$

Since $(1-k_{SB})$ and $(k_{ST}-1)$ are both positive it easily follows from this equation that the three values λ'_S, λ''_S and λ'''_S must either be all equal or all unequal, and that the indirect value λ'''_S must be intermediate between the two direct values, whatever the nature of the experimental errors*. The practical significance of various possible errors is examined in more detail in section 4 b).

e) Determination of elastic constants

The elastic constants utilised in the investigation were measured in the Strength of Materials Section of the Basic

* We ignore cases where either k_{SB} or k_{ST} is so close to unity that experimental errors might cause a change of sign of $(1-k_{SB})$ or $(k_{ST}-1)$ since such conditions would not be acceptable as a basis for the similarity method.